

A hybrid approach to assess systemic risk in financial networks

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We propose a credit risk approach in which financial institutions, modelled as a portfolio of risky assets characterized by a probability of default and a correlation matrix, are the nodes of a network whose links are credit exposures that would be partially lost in case of neighbours' default. The systemic risk of the network is described in terms of the loss distribution over time obtained with a multi-period Montecarlo simulation process, during which the nodes can default, triggering a change in the probability of default in their neighbourhood as a contagion mechanism. In particular, we have considered the expected loss and introduced new measures of network stress called PDImpact and PDRank. They are expressed in monetary terms as the already known DebtRank and can be used to assess the importance of a node in the network. The model exhibits two regimes of 'weak' and 'strong' contagion, the latter characterized by the depletion of the loss distribution at intermediate losses in favour of fatter tails. Also, in systems with strong contagion, low average correlation between nodes corresponds to larger losses. This seems at odds with the diversification benefit obtained in standard credit risk models. Results suggest that the credit exposure network of the European global systemically important banks is in a weak contagion regime, but strong contagion could be approached in periods characterized by extreme volatility or in cases where the financial institutions are not adequately capitalized.

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I. INTRODUCTION

One of the lessons that has been learned from the recent credit crisis is that the stability of the financial system cannot be pursued focussing exclusively on each individual bank or financial institution. A broader approach - macroprudential policy- is required, as interconnections and interactions are at least as important in contributing to the overall dynamics [16][20][10][14][13]. In order to limit systemic risk, defined as the risk that a considerable part of the financial system is disrupted, a number of dedicated boards and committees have been created: the Financial Policy Committee (FPC) at the Bank of England, the European Systemic Risk Board (ESRB) and the Financial Stability Oversight Council in the United States. The regulators are looking at new methodologies and ideas from different disciplines as it is felt that the existing models have shown serious limitations, being incapable of predicting the timing and the extent of the financial crisis [21]. In particular techniques borrowed from network science [28] [6] have been successfully applied to the study of network resilience to external shocks [7] [29] and have proven useful in the analysis of financial systemic risk [17] [12] [22] [5] [27].

In this context, financial institutions are described as nodes in a network, connected by different kinds of edges, indicating: cross ownership [31], investments in the same set of assets (overlapping portfolios) [32] or credit exposures [8] [26]. Another natural way to approach the problem is to consider the financial institutions as a portfolio

of risky assets with assigned probability of default and correlations. In this context the risk is thought as the premium to insure the portfolio against the loss of a percentage of the total asset amount [18]. The framework we propose in this article aims at combining the two approaches, using all the available information about the system. The original models used to study network stability [11] relied on a variant of the 'domino effect' to propagate the stress and, if the original shocks was not big enough to start the chain reaction, no quantifiable effect could be calculated. To overcome this limitation Battistoni et al [4] introduced DebtRank, a new measure of systemic risk. The DebtRank of node i , is a number measuring the fraction of the total economic value in the network that is potentially affected by the distress or the default of node i . The measure presented interesting characteristics such as being expressed in monetary terms and being able to 'feel' the stress in the network also in absence of actual defaults. However, it is not evident how, in the real world, the propagation of the stress postulated by the model would happen and how it would translate in an actual loss for the banks. In order to fill this gap Bardoscia et al [2] proposed a slightly modified model and a derivation of the dynamics for the shock propagation using basic accounting principles. To obtain their results, the authors had to make the not fully financially justified assumption that the exposures towards other banks lose their value proportionally to the loss in equity suffered by the borrowing banks, namely:

$$a_{ij}(t+1) = a_{ij}(t) \frac{E_j(t)}{E_j(t-1)} \quad (1)$$

where $E_j(t)$ and $a_{ij}(t)$ are, respectively, the equity of bank j and the exposure that bank i has with bank j at

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time t . The above updating equation is used when bank j has not defaulted in the previous time period, otherwise $a_{ij}(t+1)$ is set to be zero. In such approach it is also crucial to understand how the time step is defined: is it a year, a quarter or a minute? The answer is not irrelevant because one of the findings of Ref. [2] is that, no matter how small the initial shock is, if the modulus of the largest eigenvalue of the interbank leverage matrix $\Lambda_{ij} = \frac{a_{ij}}{E_i}$ is greater than one, at least one bank fails. This is the equivalent of the 'butterfly effect' for banks which is clearly unrealistic in actual financial networks. The main issue relative to Eq. (1) is that it fails to consider the stochastic nature of the potential loss associated to nodes with diminished equity. We believe that the most useful objects of investigation are the probabilities of defaults of the nodes and not the assets. In introducing our model, we maintain the characteristics of DebtRank: a monetary value for the 'centrality measure' of a node and the sensitivity to a distress of the network also in absence of actual default. In addition to this, our model uses techniques that are familiar to finance professionals [25] and its contagion mechanism is intuitive, with the default of a node increasing the probability of default of its neighbourhood. The introduction of a well-defined unit of time allows analysing the dynamic of stress propagation and modelling the financial institutions as complex agents reacting to evolving macroeconomic scenarios.

II. THE PD MODEL

In our model, financial institutions are described as the nodes of a network. Each node i , with $i = 1, 2, \dots, N$, is characterised, at time t , by the total asset $A_i(t)$, i.e the set of anything a financial institution owns and that can be converted to cash, by a loss given default $LGD_i(t)$, representing the percentage of the total asset that would be lost in case of default, and by a threshold $E_i(t)$, denoting the capital of i , more precisely the "tier1 capital", which is the capital of the bank that can be used to absorb losses. Furthermore, we associate to each financial institution i a probability of default per time interval $PD_i(t) \equiv PD_i(t, \Delta t)$. This is a crucial quantity of our model, and represents the probability that node i defaults in the time interval $[t, t + \Delta t]$. The links of the network are directed and weighted, and represent credit relationships, for example loans, among financial institutions. In practice, we set the weight a_{ij} of the link from i to j equal to the exposure of i to the default of node j .

In order to quantify systemic risk, we use a multi-period simulation process with M time iterations, during which the nodes can default, triggering an increase of the probability of default in their neighbourhood as a contagion mechanism. The time step Δt is chosen coherently with the definition of $PD_i(t, \Delta t)$, hence it is a well defined unit of time. In the following computations we will use $\Delta t = 1$ year. In our model, the monetary impact on node i due to the defaults of neighbouring nodes is

quantified as:

$$I_i(t) = \sum_j a_{ij}(t) \delta_j(t) LGD_j(t) \quad (2)$$

where $\delta_j(t)$ is equal to 1 if node j has defaulted at time t , and is 0 otherwise. We will consider $a_{ij}(t) = a_{ij}$ as a constant and the index j in the sum includes all the nodes that had not been defaulted at the previous times $0, \dots, t - \Delta t$. The monetary impact on node i would, in turn, trigger a modification of the total assets and the thresholds, but also of the probabilities of default, of node i at the next time, according to the following updating equations:

$$\begin{aligned} E_i(t + \Delta t) &= \max[0, E_i(t) - I_i(t)] \\ A_i(t + \Delta t) &= \max[0, A_i(t) - I_i(t)] \\ PD_i(t + \Delta t) &= f[PD_i(t), I_i(t), E_i(t), A_i(t), \dots] \end{aligned} \quad (3)$$

where $PD_i(t + \Delta t)$ is in general a function f of the probability of default $PD_i(t)$, the impact experienced, the threshold and the total asset at the previous time. In this paper we will not take into account the dynamics of $LGD_i(t)$. We will set $LGD_i(t) = 0.6$ for each bank i to be constant in time.

We have established a mapping between probability of default and change in equity in the context of standard financial risk management theory [3] using the Merton model [23]:

$$PD_i(t) = 1 - \Phi\left(\frac{\ln A_i(t) - \ln B_i - 0.5\sigma_i^2}{\sigma_i}\right) \quad (4)$$

where Φ is the cumulative gaussian distribution and, for simplicity, we have set the drift term equals to zero and considered a period of one year (see Materials and Methods). The total asset volatility σ_i and the total liability B_i are considered constant during the simulations, with:

$$B_i = A_i(0) - E_i(0) \quad (5)$$

Hence $A_i(t)$ can be expressed as $A_i(t) = E_i(t) + B_i$, and Eq. (4) provides an updating equation for $PD_i(t + \Delta t)$ once the right hand side is updated with the new value of $E_i(t + \Delta t)$ obtained from Eqs. (3). We also set $PD_i(t + \Delta t) = 1$ when the impact $I_i(t)$ on the financial institution i is equal to its tier1 capital $E_i(t)$, i.e to the maximum monetary loss the institution can tolerate (see Fig. 1). The procedure that we have just described, for updating the probabilities of default will be referred to as the *Merton update*. We have also explored a *Linear update*, i.e. a linear relationship between $PD_i(t + \Delta t)$ and the impact $I_i(t)$:

$$PD_i(t + \Delta t) = \min\left[1, PD_i(t) + \frac{(1 - PD_i(t))I_i(t)}{E_i(t)}\right] \quad (6)$$

with $PD_i(t + \Delta t)$ being capped to 1 when the impact $I_i(t)$ is greater or equal to $E_i(t)$ (see Fig. 1).

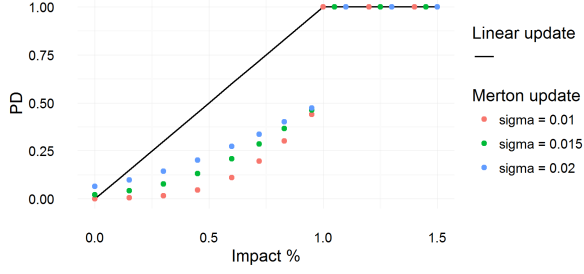


FIG. 1: Probability of default PD of a node as a function of the impact I expressed as a fraction of the capital E . When the ratio I/E is equal or greater than 1 we set $PD = 1$ as the financial institution is insolvent and it will default during the next time period. The continuous line describes the Linear update while the dots represent the Merton update with different values of asset volatilities σ .

In our simulations, at each of the M time periods, the defaults are driven by a stochastic process (X_1, X_2, \dots, X_N) which is assumed to follow a multivariate gaussian distribution with a correlation matrix whose elements $\{\rho_{i,j}\}$ are, for instance, calculated from the equity returns of financial institutions i and j , obtained from the stock market quotations [19]. According to Ref. [18] we consider $\rho_{i,j} = \rho \quad \forall i, j$. In order to reflect the fact that the average correlation increases during periods of crises, it is possible to use updating equations such as:

$$\rho(t + \Delta t) = \min[1, \rho(t) + N_D(t) * \Delta_C(t)] \quad (7)$$

for the pairwise coefficient ρ , where $N_D(t) = \sum_j \delta_j(t)$ is the number of defaulted nodes at time t . The quantity $\Delta_C(t)$ represents the average correlation increase per single default at time t and needs to be calibrated (or assumed in a stress scenario). For simplicity, we have not used Eq. (7) but we have performed simulations with different values of ρ (kept constant during the simulation).

The default of financial institution i at time t (more specifically in the temporal window $[t, t + \Delta t]$) happens when the corresponding drawn value x_i of random variable X_i in the sampling $(X_1 = x_1, X_2 = x_2, \dots, X_N = x_N)$ is smaller than $\Phi^{-1}(PD_i)$, with Φ being the cumulative gaussian distribution. If at least a node has defaulted at time t , we update the variables of the system for the next time $t + \Delta t$ as in Eqs. (3). Defaulted nodes are then removed. Instead, if no node has defaulted at time t , we proceed to the following time step and the new sampling $(X_1 = x_1, X_2 = x_2, \dots, X_N = x_N)$ with the same network and stochastic process parameters. The simulation is then continued for M temporal iterations.

The loss $L(t)$ for the entire network at time t is calculated as:

$$L(t) = \sum_j A_j(t) LGD_j(t) \delta_j(t) \quad (8)$$

while the total loss L_{tot} is obtained by summing up the

discounted values of the losses at the different time periods:

$$L_{\text{tot}} = \sum_{t=1}^M L(t) D(t) \quad (9)$$

where $D(t)$ is the discount factor relative to time t (in this paper we set $D(t) = 1$ for each t). We have considered an ensemble of 100000 different Montecarlo realizations of the $M = 7$ iterations of the stochastic process corresponding to a total period of 7 years, and we have calculated distributions and averages of different quantities, such as the losses, over the ensemble. We indicate with the symbol $\bar{\bullet}$ such averages. The fact that our Montecarlo simulations involve a number of time steps and in each of them the contagion is spread via a modification of the probability of default, is a crucial difference with respect to standard credit risk management techniques.

A. Stress scenario: PDImpact and PDRank

In our framework, the nodes are characterized by an initial probability of default $\mathbf{PD}(t) \equiv (PD_1, PD_2, \dots, PD_N)$ at time $t = 0$. Hence, even in absence of any external shock, the system can suffer losses during the simulations within the considered time frame of M time periods. The loss distribution so obtained, and in particular the expected loss $\bar{L}_{\text{tot}}(\mathbf{PD})$ can be used as the base-line for comparison with the losses in presence of stress. Since a distress of the network is described as an increased probability of default of a set of nodes, $\delta\mathbf{PD}$, we can introduce the so-called *Probability of Default Impact* ($PDImpact$), indicated as $C(\delta\mathbf{PD})$, of the stressing perturbation $\delta\mathbf{PD}$ onto the initial probability of default \mathbf{PD} as:

$$C(\delta\mathbf{PD}) = \bar{L}_{\text{tot}}(\mathbf{PD} + \delta\mathbf{PD}) - \bar{L}_{\text{tot}}(\mathbf{PD}) \quad (10)$$

where the two terms on the right hand side are respectively the average loss of the network in the presence and absence of the additional stress $\delta\mathbf{PD}$.

Analogously, we can also introduce a node centrality measure, that we name the *Probability of Default Rank*, or $PDRank$, for assessing the relative importance of each financial institution. The $PDRank$ of node i is obtained multiplying the probability of default of node i by the additional average loss experienced by the network due to the default of node i :

$$PDRank_i = PD_i \cdot \left(\bar{L}_{\text{tot}}(\mathbf{PD}^{D_i}) - \bar{L}_{\text{tot}}(\mathbf{PD}^{I_i}) \right) \quad (11)$$

where \mathbf{PD}^{D_i} is the initial probability vector in which the probability corresponding to node i has been set to 1 at $t = 0$, while \mathbf{PD}^{I_i} is the initial probability vector where the probability corresponding to node i has been set to 0 and kept at the value 0 for each time $t \geq 0$ (the node cannot default during the simulation). Therefore, the

quantities $\bar{L}_{\text{tot}}(\mathbf{PD}^{Di})$ and $\bar{L}_{\text{tot}}(\mathbf{PD}^{Ii})$ represent respectively the average loss, during the simulation, when node i defaults at time 1, and when node i cannot default (the average loss that the network would suffer anyway irrespective of the node i). In practice, $PDRank_i$ of node i measures the expected loss “due” to node i . As the already known DebtRank, it is expressed as a monetary value and can be used to rank the nodes in terms of their ‘systemic risk’.

A further characterization of a network, which we name $PDBeta$, can be obtained by quantifying the sensitivity of the system to a percentage increase of all the initial probabilities of default. Assuming an approximate linear relationship between the PDImpact $C(\delta\mathbf{PD}^*)$ obtained for an increase of the probabilities of default $\delta\mathbf{PD}^* \equiv \mathbf{PD} \cdot x/100$ and the percentage of increase x , we can define $PDBeta$ as follow:

$$PDBeta = \frac{C(\delta\mathbf{PD}^*)}{x} \quad (12)$$

In this way $PDBeta$ represents the variation of PDImpact for a unitary percentage variation of the probabilities of default.

Results

We have applied our model to analyse the data collected by the European Banking Authority (EBA) relative to the European global systemically important banks (gsib), and we have used a new algorithm to infer the network of exposures from the incomplete data provided (see Materials and Methods). The initial values of the probabilities of default have been obtained from public information about the credit rating of the banks and from statistics available on the Fitch website (see Materials and Methods). The results in this section should be considered as an academic exercise to illustrate the characteristics of the model as we do not have the exact adjacency matrix of exposures and a number of approximations and assumptions have been made. Also it is clearly unreasonable assuming that the banks or the regulators would not react after the first defaults (replenishing their tier1 capital for example). In the following, where not specified we use an average correlation $\rho = 0.5$.

B. Two ‘regimes’: weak and strong contagion

The distributions of the total loss L_{tot} experienced by the network under different values of average correlations ρ and different values of the thresholds are shown respectively in Fig. 2 and Fig. 3. We say that the system is in a ‘strong contagion’ (as opposed to ‘weak contagion’) regime if its ability to propagate the stress on the network affects the loss distribution, depleting the intermediate losses in favour of the extreme end of the tail.

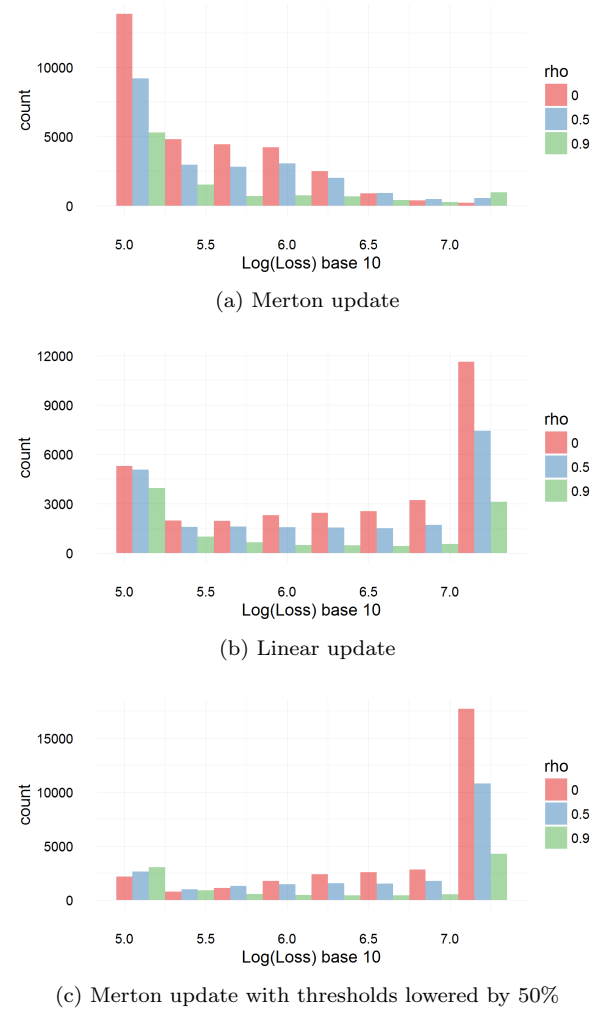


FIG. 2: Distributions of the network total loss for different values of the average correlation ρ . In panel (a), higher correlations increase the losses with the Merton update model. This is in contrast with the Linear update reported in panel (b), where higher correlations correspond to lower losses. Panel (c) shows a similar behaviour as panel (b), using the Merton update but reducing all the thresholds by 50 %.

Fig. 2(b) and and Fig. 3(b), relative to the Linear update, present strong contagion characteristics. In particular Fig. 2(b) shows that lower average correlation corresponds to more pronounced losses as it is more likely to have at least one default during the first time periods and subsequently the loss is amplified by the contagion. The Merton update model in Fig. 2(a) does not show strong contagion effects with the available data. This does not mean that they cannot appear under particular circumstances. For example, the distribution corresponding to thresholds reduced by 50% or 90% in Fig. 3(a) clearly shows the depletion of the loss distribution in the middle with a marked increase in the tail. As a further example, as shown in Fig. 2(c), when the thresholds are reduced at half of their values, also the system with Merton update shows increasing losses at lower value of average ρ .

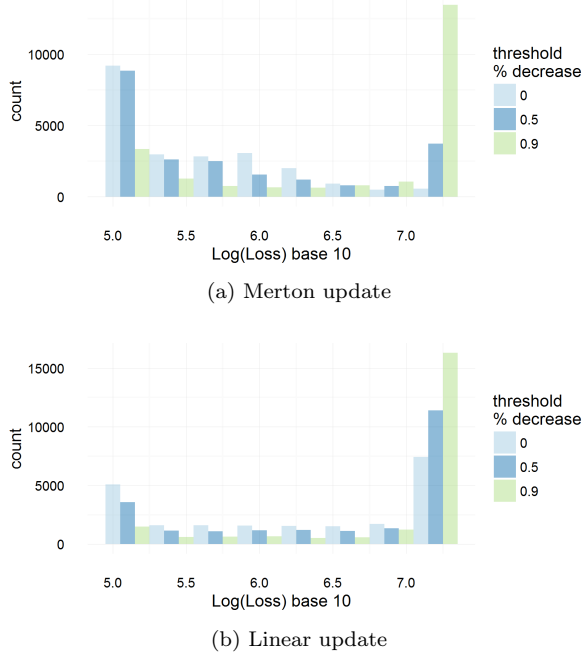


FIG. 3: Distributions of the network total loss for different values of the thresholds. When the thresholds are reduced by 50% or 90% the loss distributions accumulate on the far right of the tail both for the Merton update in panel (a) and for the Linear update in panel (b)

Fig. 4 confirms an approximate linear relationship between PDImpact and a percentage increase of the initial probabilities of default, allowing the definition of the measure $PDBeta$ (Eq. (12)) that can be used, together with the expected loss \bar{L}_{tot} , to gauge the riskiness of a network. Defining the global asset of the network as $A_{glob} = \sum_{k=1}^N A_k$, we have found $\bar{L}_{tot}/A_{glob} = 0.93\%$ and $PDBeta/A_{glob} = 0.0124\%$ for the Merton update, and $\bar{L}_{tot}/A_{glob} = 5.125\%$ and $PDBeta/A_{glob} = 0.0318\%$ for the Linear update.

C. The critical nodes of the network

We can now analyse the relative contribution of the nodes to the systemic risk of the network. Table I reports the values of PDRank in million of EUR, obtained using respectively the Merton and the Linear update. The ranking of the most important nodes is different in the two cases and the corresponding values can vary by more than one order of magnitude for nodes with a high probability of default such as BFA with $PD = 0.0116$ and MPS with $PD = 0.0093$. These nodes can act as a catalyst for a chain reaction of losses especially in a 'strong contagion' regime: relatively small losses can have a dramatic effect on the probability of default of the impacted nodes and this explains why they are at the top of the DebtRank table in the Linear update case. The ranking implied by PDRank is different from the one that takes

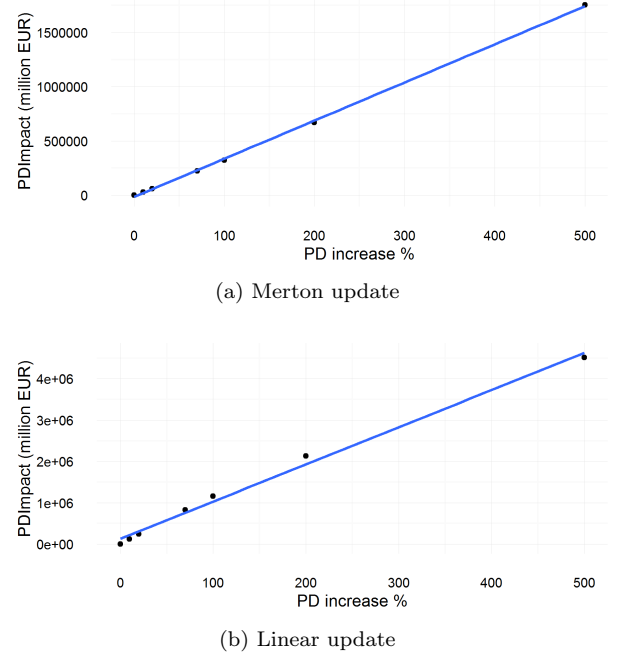


FIG. 4: Approximate linear dependence between PDImpact $C(\delta PD^*)$ obtained for an increase of the probabilities of default $\delta PD^* \equiv PD \cdot x/100$ and the percentage of increase x . In the analysed network, the average loss increase per 1% increase in the probability of default is about 3.5 and 9 billion EUR respectively for the Merton and for the Linear update.

into consideration the total asset of the financial institutions (as in a 'too big to fail' approach). This is evident as the PDRank definition includes the probability of default, which is not related to the total asset. We can investigate if PDRank can be explained by the probability of default multiplied by the total asset. Fig. 5 shows that this is not the case even if there is definitely a positive correlation. It is interesting to note that BFA and MPS are well above the regression line in case of the Linear update (Fig. 5b), which reflects once again the increased role of the probability of default in a strong contagion regime.

III. CONCLUDING REMARKS

The model described in this paper uses different types of information regarding the financial institutions: correlation structure, probabilities of default and network of exposures. It allows following the evolution in time of the network under different stress scenarios, using multi period Montecarlo simulations. In this paper we focused our attention to average quantities to characterized the riskiness of the network, such as expected loss and the slope of the expected loss vs a percentage increase of the probabilities of default ($PDBeta$). However our model allows obtaining the entire loss distribution and other statistics (e.g. quantiles) could be more appropriate for

PD	Threshold	Total Asset	Bank	PDRank
0.001	70378	2252752	BNP Paribas	10784
0.001	59081	1940282	Barclays	6728
0.0017	45499	1034421	Unicredit	4866
0.001	51250	1410547	RBS	3430
0.001	25123	655686	Commerzbank	3254
0.001	70719	1723006	Credit Agricole	3003
0.001	64250	1455593	Santander	2759
0.001	63397	1659337	Deutsche Bank	2730
0.0116	11879	234816	BFA	2139
0.001	49969	1336600	BPCE	1962
0.0093	6608	201385	MPS	1919

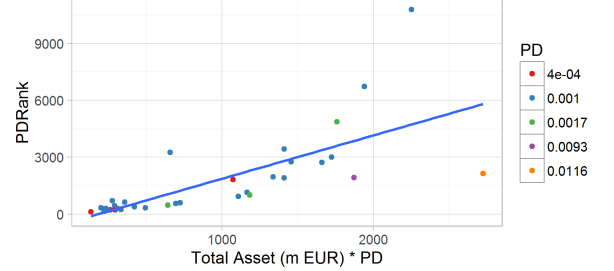
(a) PDRank - Merton update

PD	Threshold	Total Asset	Bank	PDRank
0.0093	6608	201385	MPS	82209
0.0116	11879	234816	BFA	75002
0.0017	45499	1034421	Unicredit	26010
0.0017	38247	695873	Intesa Sanpaolo	20507
0.001	70378	2252752	BNP Paribas	15636
0.001	59081	1940282	Barclays	15512
0.001	51250	1410547	RBS	15393
0.001	63397	1659337	Deutsche Bank	15313
0.001	25123	655686	Commerzbank	15311
0.001	64250	1455593	Santander	15217
0.001	70719	1723006	Credit Agricole	15204

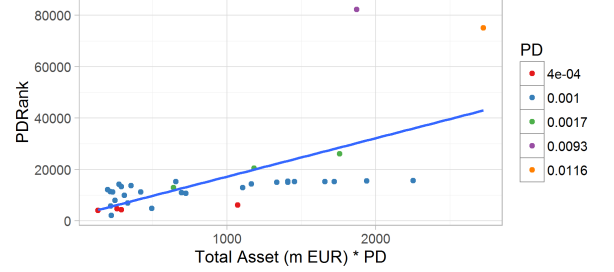
(b) PDRank - Linear update

TABLE I: Top twelve nodes ordered by PDRank in the network of global systemically important banks of the European Union. The data is relative to the end of 2014 which is time 0 in our simulations.

particular investigations. The nodes can be ranked in a natural way with respect to their contribution to the systemic risk using the centrality measure 'PDRank'. The contagion propagation is easy to understand as it is linked to an increase of the probability of default of a node due to the default of its neighbours. We have also introduced an algorithm to re-create a network of individual exposures given the information of the total intra-node assets and liabilities for each node. A 'strong contagion' regime has been identified, characterized by larger losses at lower average correlation between nodes and by a depletion of the loss distribution at intermediate losses in favour of a fatter tail. The financial network considered does not present strong contagion effects when using the Merton update. However we have seen that such effects can appear when the banks are not sufficiently capitalized (i.e. thresholds are low). Furthermore we have kept the asset volatility σ constant in the Merton model. In periods with extreme volatility, σ would be bigger and the PD calculated with the Merton update (Eq. (4)) could increase significantly, moving the system toward the strong contagion regime.



(a) Merton update



(b) Linear update

FIG. 5: PDRank of a financial institution is shown as a function of the probability of default of the corresponding node times its total asset. Panel (a) and (b) refer to the Merton and the Linear update respectively. While there is a positive correlation, PDRank cannot be explained completely with a linear regression and the differences can be thought of as due to network effects.

IV. MATERIALS AND METHODS

European global systemically important banks.

As part of its mandate, the European Banking Authority collect data annually from the global systemically important banks (gsib) in the European Union and publish the results in their website 'www.eba.europa.eu'. We have chosen the data from the year 2014. The data set contains the fields 'Intra-financial system assets' and 'Intra-financial system liabilities' that we use in our model to recreate the individual exposures using the algorithm described in the next paragraph. The field 'Total exposures' provides a proxy for the total assets ($A_i(t)$). The tier1 capital has been obtained from another study performed by EBA in cooperation with European Systemic Risk Board (ESRB): 'The EU-wide stress test', that aims at 'assessing the resilience of financial institutions to adverse market developments'. We have selected the Banks that were in both exercises and we identified 35 institutions. The initial probability of default has been obtained from the table 'Financial Institutions Average Annual Transition Matrix: 1990-2014' in the document '2015 Form NRSRO Annual Certification' obtained from Fitch website www.fitchratings.com.

Inferring the network. We describe a new algorithm to infer the network from incomplete data. In the literature, a maximum entropy algorithm [30] has often

been used but it is known that it might not represent the best choice for recreating a realistic interbank network [24] and different alternatives have been proposed [8][15][9]. We want to capture the fact that small financial institutions are more inclined to have connections with a small number of bigger banks. The level of exposure tends to be above a certain minimum value as the creation of a credit relationship involves a 'maintenance' cost. This was already addressed by Anand et al. [1] but here we propose an alternative algorithm that we find more intuitive and that allows controlling over the minimum exposure amount and the 'degree of attraction' between smaller nodes and bigger ones. The main idea is to match asset with liabilities, building the adjacency matrix in steps: 1) The smaller borrower nodes choose first where to get the money from; 2) The lender (a different node) is chosen randomly with a probability that is proportional to its remaining assets to the power of alpha (alpha being the parameter for tuning the degree of attraction between heterogeneous nodes). 3) The loan amount is chosen as a percentage of the total liabilities of the borrower node and represents the minimum exposure that it is convenient to exchange, constrained by the 'residual' assets of the lender and the 'residual' liabilities of the borrower. 4) The adjacency matrix and the residual asset and liabilities amount are updated. 5) The process continues till all the assets are matched with all the liabilities. 6) If at the end remains one node that can borrow money only from itself, the procedure re-routes some of the previous loans so that the adjacency matrix

is completed with zero values on the diagonal.

Merton Model. The Merton model can be used to evaluate the probability of default of a company i described as having a single liability $B_i(t) = A_i(t) - E_i(t)$ in terms of a bond issued and expiring at time $T = t + \Delta t$. The asset A_i of the company is thought as following a log-normal random process with drift μ_i and volatility σ_i . A default occurs if during a simulation the asset $A_i(t)$ falls below the value $B_i(T)$ at time T . Assuming $B_i(t) = B_i$ as a constant, it is possible to calculate the probability of default as:

$$PD_i(t) = 1 - \Phi \left(\frac{\ln A_i(t) - \ln B_i + (\mu_i - 0.5\sigma_i^2)\Delta t}{\sigma_i\sqrt{\Delta t}} \right) \quad (13)$$

Φ is the cumulative gaussian distribution. To simplify we assume $\mu_i = 0$ and $\Delta t = 1$ year. Having $PD_i(0)$, $A_i(0)$ and $E_i(0)$ it is possible to calculate B_i and, inverting Eq. (13), σ_i . We make the assumption that σ_i doesn't change during the simulation. It is then possible to use Eq. (13) to update the probability of default $PD_i(t)$ given the new values of $A_i(t)$ and $E_i(t)$. The assumption of constant σ_i is not completely satisfactory as it is reasonable to expect that the volatility increases when the company approaches the default. It is possible to devise a more complex implementation of the model that includes a dynamics for $\sigma_i(t)$.

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